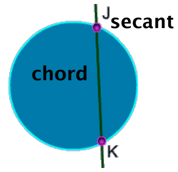
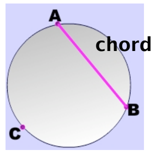
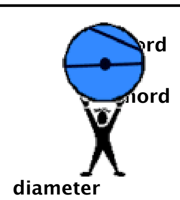
Circles- Chords, Secants and Tangents

Circles and Chords







A __________________ is a segment that joins two points of the circle.

A __________________________ is a ___________ that contains the __________________________________ of the circle.

A __________________________ is a line that intersects a circle in ________ places and continues through the circle. A secant _____________________ through a circle.

Theorems:

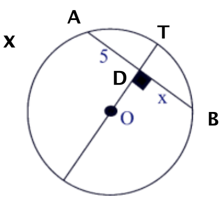
1. In a circle, a radius ______________________________ to a chord _________________ the chord.

2. In a circle, a radius that __________________ a chord is _______________________________________ to the chord.

3. In a circle, the _______________________________________ of a chord passes through the ______________ of the circle.

Problems using Theorem 1, 2 and 3:

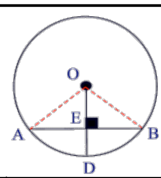
Find x



Proof of Theorem 1:

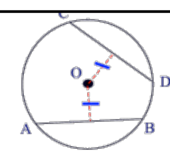
Given: $\odot O$, $\overline{OD} \perp \overline{AB}$

Prove: $\overline{OD}$ bisects $\overline{AB}$



$\odot O$, $\overline{OD} \perp \overline{AB}$	Given
Draw $\overline{OA}$, $\overline{OB}$	Two points make a line
$\angle OEA$, $\angle OEB$ are right angles	⊥ lines form right angles
$\triangle OEA$, $\triangle OEB$ are right triangles	right ⊿s contain one right angle
$\overline{OA} \cong \overline{OB}$	
$\overline{OE} \cong \overline{OE}$	
$\triangle AOE \cong \triangle BOE$	
$\overline{AE} = \overline{BE}$	
E is the midpoint of $\overline{AB}$	midpoint divides into ≅ parts
$\overline{OD}$ bisects $\overline{AB}$	bisector intersects at midpoint

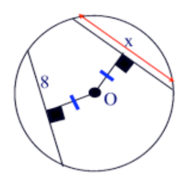
Theorem 4:



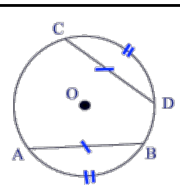
In a circle, or congruent circles, ____________________________ chords are _________________________ from the center.

Problems using Theorem 4:

Find x.



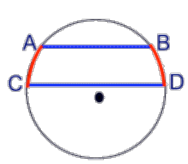
Theorem 5:



In a circle, or congruent circles, congruent ________________ have congruent __________________.

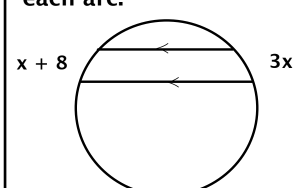
Theorem 6:

In a circle, ____________________________________ intercept congruent ___________. Note, the ____________________ are not necessarily congruent, just the ______________ are.



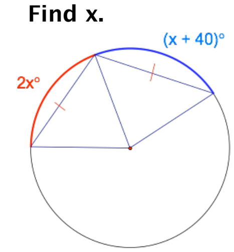
Problems using Theorems 5 and 6:

Find the measure of each arc.



Solution:

Find x.



Solution:

Circles- Chords, Secants and Tangents

Closing: Complete the chart.

Theorem	Summary
1	a radius ⊥ to a chord _________________ the chord.
2	a radius that bisects the chord is __________________ to the chord and they will therefore meet at ______________________ angles.
3	the _______________ bisector of a chord can help you find the _____________________ of the circle.
4	If two chords are equidistant from the center of a circle, they are ___________________________________
5	congruent chords have congruent _______________
6	If two chords are parallel, the two _______________ between them are congruent.

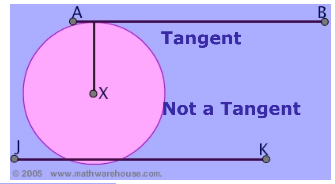
Homework Week 3: (Due 1/29/09)

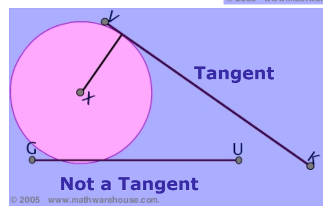
Complete Set A.

Tangent of a Circle

A tangent has two defining properties

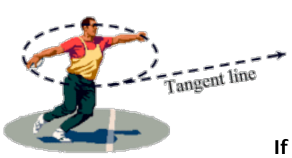
- **A tangent touches a circle in ______________________**
- **The tangent intersects the circle's radius at a __________ ______________________**



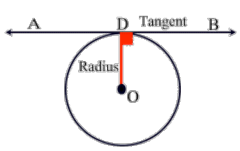


Tangents and Circles

A tangent to a circle is _____________ in the plane of the circle that ____.



If you spin an object in a circular orbit and release it, it will travel on a path that is tangent to the circular orbit.



If a line is tangent to a circle, it is _________________________ to the _________________ drawn to the point of tangency.

IF: $\overline{AB}$ is a tangent

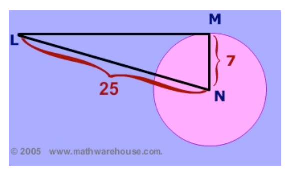
D is point of tangency

THEN: $\overline{OD} \perp \overline{AB}$

Example 1:

What must be the length of LM for this segment to be tangent line of the circle with center N?

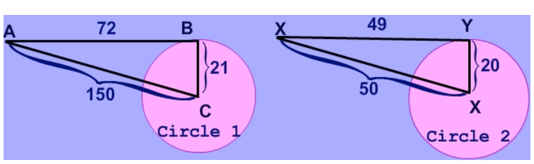
Because the tangent is perpendicular (meets at a ______________________) to the ____________, we can use the ____ to find the length of LM.



Determining if a line is a tangent

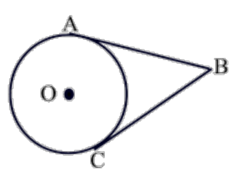
Because the tangent is perpendicular to the radiius, we can use the Pythagorean Theorem to determine if a line is a tangent. If we do not get a true statement using the Pythagorean Theorem, the line is NOT a tangent.

How many, if any, of the circles above have tangent line? In both cases X is the center of the respective circles.



Theorem:

Tangent segments to a circle from the _______________ __________________ are ____________________________.

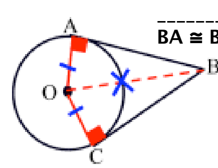


IF: $\overline{AB}$ is a tangent to circle O at A

$\overline{CB}$ is a tangent to circle O at C

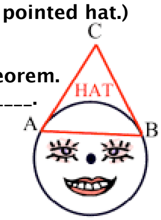
THEN: $\overline{AB} \cong \overline{CB}$

(You may think of this as the "Hat" Theorem because the diagram looks like a circle wearing a pointed hat.)



OAB ≅ OCB by the ______________________ theorem.

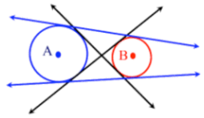
BA ≅ BC by ________________.



Circles- Chords, Secants and Tangents

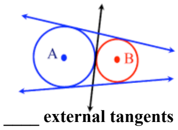
Common Tangents: Common tangents are lines or segments that are tangent to more than one circle at the same time.

4 Common Tangents
(2 completely separate circles)



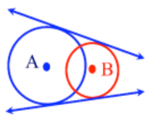
_____ external tangents (blue)
_____ internal tangents (black)

3 Common Tangents
(2 externally tangent circles)



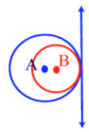
____ external tangents (blue)
_____ internal tangents (black)

2 Common Tangents
(2 overlapping circles)



_____ external tangents (blue)
______ internal tangents (black)

1 Common Tangents
(2 internally tangent circles)

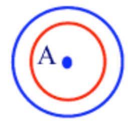


_____ external tangents (blue)
_____ internal tangents (black)

The only ways to have no Common Tangents:

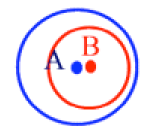
0 Common Tangents
(2 completely separate circles)

(2 concentric circles)
Concentric circles are circles with the same center.



0 _________________ tangents
0 _________________ tangents

(one circle floating inside the other, without touching)



0 _______________ tangents
0 _______________ tangents

Closing: Complete the chart.

Def.	Summary
1	tangents intersect a circle in _________ point.
2	a tangent is ____ to the radius of the circle.
3	Tangent segments to a circle from the same external point are__________________.
4	Because tangents are perpendicular to the radius, we can use the ___________________________________ to find missing lengths.
5	circles can have up to _______ common tangents if they do not intersect and are not inside one another.
6	Between two circles there can be _________________ tangents and _________________ tangents.

Homework Week 3: (Due 1/29/09)

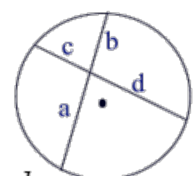
Complete Set B.

Rules for Dealing with Chords, Secants, Tangents in Circles

Theorem 1:

If two chords intersect in a circle, the _____________________ of the lengths of the _____________________ of one _________ equal the ____________________ of the segments of the other.

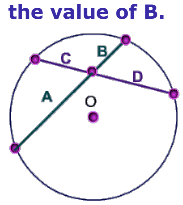
Intersecting Chords Rule:
(segment piece)•(segment piece) =
(segment piece)•(segment piece)



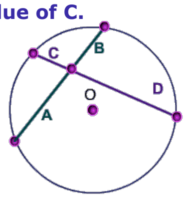
$$a \cdot b = c \cdot d$$

Example:

In the circle below, the chord segments have the following lengths: A= 6, C=3, D=4. Use the theorem for the product of chord segments to find the value of B.

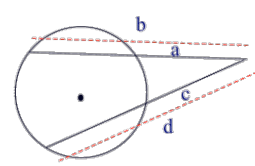


In the circle below, the chord segments have the following lengths: A= x + 4, B=3, D= 6 . Use the theorem for the product of chord segments to find the value of C.



Theorem 2:

$$a \cdot b = c \cdot d$$



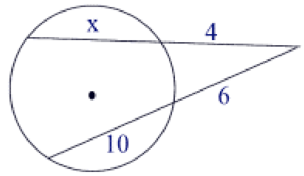
Secant-Secant Rule:
(whole secant)•(external part) =
(whole secant)•(external part)

If two secant segments are drawn to a circle from the ______________ ________________________, the product of the length of __________ ___________________ segment and its ________________________ part is equal to the product of the length of the other _______________ segment and its _____________________ part.

Circles- Chords, Secants and Tangents

Example 1:

Solving for x using the Secant-Secant Rule



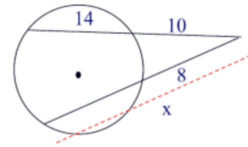
Secant-Secant Rule:
(whole secant)•(external part) =
(whole secant)•(external part)

whole secant: ___________
external part: _______

whole secant: ___________
external part: _______

Example 2:

Solving for x using the Secant-Secant Rule

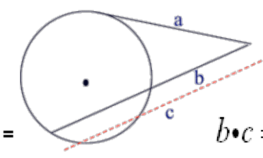


Secant-Secant Rule:
(whole secant)•(external part) =
(whole secant)•(external part)

whole secant: ___________
external part: __________

whole secant: _______
external part: _______

Theorem 3:



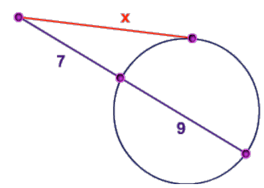
$$b \cdot c = a^2$$

Secant-Tangent Rule:
(whole secant)•(external part) =
$(\text{tangent})^2$

If a secant segment and tangent segment are drawn to a circle from the _____________________________, the product of the length of the ___________ segment and its external _________ equals the ______________ of the length of the ____________________ segment.

Example:

Secant-Tangent Rule:
(whole secant)•(external part) =
$(\text{tangent})^2$



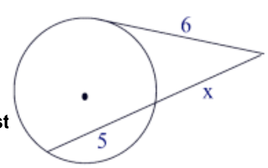
In the following problem, the red line is a tangent of the circle, what is its length?

secant: __________________
external part: __________

tangent: ______________

Example:

You may have to solve tangent problems by factoring a quadratic equation.



__________- First, Outer, Inner, Last
Remember, the product of two binomials is a quadratic equation.

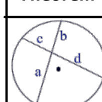
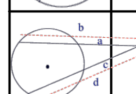
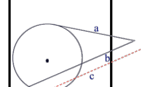
The middle term is formed by the __________ of the outer and inner products.

What is its length of the external part of the secant?

whole secant: __________
external part: _________

tangent: __________

Closing: Complete the chart.

Theorem	Summary
	If two chords intersect in a circle, the _________ of the segments of one chord equal the ________________ of the segments of the other.
	If two secant segments intersect the same external point, the product of one ____________ segment and its external _________ equals the product of the other ______________ segment and its external _______________.
	If a secant and tangent segment are intersect the same external point, the product of the ________________ segment and its external ___________ equals the _____________ of the tangent segment.

Homework Week 3: (Due 1/29/09)
Complete Set C.